A Poisson Model for Hurricanes of the North Atlantic

by

Paul M. Sommers

August 2003
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OF THE NORTH ATLANTIC

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Some are weather-wise; some are otherwise.
Benjamin Franklin
Poor Richard's Almanac

The annual number of hurricanes, which form in the North Atlantic basin, is a discrete random variable. The probability a hurricane (with a minimum sustained surface wind speed of 74 miles per hour) will form is small and here assumed to be the same throughout the season. Moreover, the number of hurricanes occurring in a given season is assumed to be independent of the number of hurricanes in any other season. Under these assumptions, is the distribution of the annual number of hurricanes that form for the entire North Atlantic basin Poisson? Surprisingly, the answer is yes.

The Poisson distribution is a discrete probability distribution which has the following formula:

\[ P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!} , \quad x = 0, 1, 2, \ldots \]

where \( \lambda \) is the mean number of occurrences or expected value of the Poisson distribution.

In Table 1, a Poisson distribution is shown to be an appropriate model for the 554 hurricanes in the North Atlantic Ocean (which includes the Caribbean Sea and Gulf of
Mexico) over the 111-year period 1886 through 1996. These data on hurricane activity are from Hebert and McAdie [Table 2, 1].

The expected value of the Poisson distribution is estimated by

$$
\bar{e} = \frac{\sum x_i O_i}{n} = 4.9909
$$

where \( n = 111 \).

How well does the observed frequency distribution conform to the Poisson distribution? The null hypothesis is that the observed or actual distribution can in fact be represented by the theoretical (Poisson) distribution, and that the discrepancies between them are due to chance. The value of the test statistic is

$$
\chi^2 = \sum_{i=1}^{11} \frac{(O_i - E_i)^2}{E_i} = 11.038
$$

Since \( \chi^2_{0.05,11} = 19.6751 \) (and exceeds the value of the test statistic), it follows that there is no reason to reject the null hypothesis.\(^2\) In other words, the probability is greater than .05 that the observed discrepancies between the actual distribution and the Poisson distribution could be due to chance.\(^3\)

We can conclude that the Poisson model provides a surprisingly excellent fit to the data on North Atlantic basin hurricanes during the period 1886-1996.
Table 1. Distribution of the number of hurricanes in the North Atlantic basin, 1886-1996

<table>
<thead>
<tr>
<th>Number of hurricanes ($x_i$)</th>
<th>Observed number of seasons ($O_i$)</th>
<th>Poisson probability ($p_i$)</th>
<th>Expected number of seasons ($E_i = 111 \cdot p_i$)</th>
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</thead>
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<tr>
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<td>0.7547</td>
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<tr>
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<td>.0339</td>
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<td>.0847</td>
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<td>19.4767</td>
</tr>
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<td>6</td>
<td>13</td>
<td>.1460</td>
<td>16.2011</td>
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<td>.1041</td>
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<td>.0034</td>
<td>0.3764</td>
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</table>
Reference

Footnotes

1. The hurricane season is defined to extend from June 1 through November 30.

2. The number of degrees of freedom is 1 less than the number of values of $(O_i - E_i)^2 / E_i$ that are summed up; that is, $13 - 1 = 12$. But it is important to note that $\chi^2$ would have 1 less degree of freedom (11, not 12), because we estimated an additional parameter of the theoretical distribution (namely, $\bar{e}$) from the actual distribution.

3. In carrying out this goodness-of-fit test, one could combine the first two intervals (0 or 1 hurricane) and the last four intervals (9, 10, 11 or 12 hurricanes) of the frequency distribution so that the theoretical or expected frequency in each and every class interval is close to being at least 5. When this is done, the calculated $\chi^2$ is 3.8367, which is still less than the critical $\chi^2_{0.05,7} = 14.0671$. 